The Properties of a Complex SIX-DIMENSIONAL Manifold

# Abstract

Here we describe a six-dimensional, pseudo-Riemannian manifold with three imaginary dimensions of time and three real dimensions of space, we assign it some trivial rules, and then describe the resulting properties. The rules are thus: any two temporal dimensions form an imaginary plane. The spatial dimensions are orthogonal to the temporal dimensions and the extent of any spatial dimension is proportional to the time coordinates and the angle between the temporal coordinate axes. We then discuss the properties that this manifold possesses when projected onto the dimensions of space and given a single evolution parameter. Here we demonstrate that the real three-dimensional projection of this complex six-dimensional manifold expands spontaneously with time and even appears to accelerate under its own power independent of any stress energy tensor.

Some formal definitions are required for our discussion.

# Real and Imaginary

For the scope of this paper, ‘real’ is anything that can be measured directly: space and acceleration for example. While ‘imaginary’ is anything that cannot be measured directly: time and velocity for example. This definition compliments the use of real and imaginary numbers in formulas presented here.

# Time

Time is an imaginary dimension. Imagine a line, give it an imaginary basis vector, an origin, and assign it components that are contravariant to the basis vector. This is time.

# Squared Time

Time, by itself, is unremarkable, but if you have two dimensions of time in an imaginary plane, then some interesting properties emerge. We can introduce a third dimension – the dimension of squared time – that is the scalar product of two imaginary coordinates. The size of this secondary dimension is:

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Where is magenta space in units of squared time. Because the spatial dimension is derived from the primary temporal dimensions, a chromatic index is used instead of a numeric one. For example, magenta is the product of red and blue. , is the coordinate in red time, , is the coordinate in blue time, and is the angle between the red and blue coordinate axes in the imaginary plane. As a convention, red time will be the coordinate axis of the observer, and blue time will be the coordinate axis (proper time) of the observed.

The magenta squared time axis is always orthogonal to the imaginary plane. The orientation of the temporal axes, , is equal to the temporal velocity, .

From Eq. (1) we see that the size of the spatial dimension of the observed object not only depends on the time coordinates, but on the temporal velocity as well. For example, the magenta squared time coordinate axis will contract as the temporal velocity between red time and blue time decreases and will vanish entirely when the coordinate axes are orthogonal (e.g., ).

Squared time has the additional property of being real. Because of this quality, the squared time projection of a complex manifold is a distinctly real submanifold and because it can be measured directly, because it is real, we can explore how this terrain changes as the imaginary coordinates evolve.

# Space

Squared time and space are the same dimension, but with different units. One cannot measure an imaginary unit like time with a real measuring stick, by definition. If we want to discuss the properties of this manifold in real numbers, then a conversion is needed.

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Where is the extent of the magenta spatial dimension in some practical unit (e.g., ), is the conversion factor between units of space and squared time (e.g., ).

A picture containing line chart

Description automatically generated

Figure 1 The relation between the primary dimensions of time, and , and a secondary dimension of space, . The size of the spatial dimension is proportional to the time coordinates and the angle between the temporal dimensions. Space is always orthogonal to time.

For the observer to objectively measure distances in the manifold, a fiducial time axis can be constructed parallel to (aligned with) the observer’s time axis. This alignment defines a reference frame and gives the observer a benchmark spatial dimension with which to compare all other orientations. By convention, blue-magenta will be the complex plane of the moving (proper) frame, while green-yellow will be the complex plane of the (coordinate) reference frame. For the reference frame, is zero.

# Velocity and Acceleration

In the special case of the reference frame, the temporal axes are aligned and can be combined into single evolution parameter, . Under these conditions, time in the two temporal dimensions advances at the same rate, the trigonometric term of Eq. (2) is unity (i.e., , ), and the spatial extent can be expressed as:

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From this, we can derive how space changes with respect to time in the reference frame in an empty manifold:

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As well as the second derivative:

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Where is the acceleration of the expansion of space and is the tangent plane (instantaneous velocity) of the manifold at time . Note that while time and velocity are imaginary, space and acceleration are real. Also note how this system of dynamics correlates to a rotation through the imaginary plane:

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| --- | --- | --- |
| Time |  |  |
| Space |  |  |
| Velocity |  |  |
| Acceleration |  | 1 |

Table 1 The relation between dynamics and the imaginary plane.

Acceleration is a critical feature of this manifold. Eq. (4) tells us that constant acceleration is manifested at every point in space independent of the coordinates. The first and most important property of this manifold is that all objects in an inertial frame will spontaneously drift away from each other at a constant rate of as the time coordinate advances. That is, objects at rest accelerate.

# Distance

Distance can be calculated using the Pythagorean theorem.

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Where is the distance, is the metric tensor and is the difference between a set of coordinates in each dimension (that is, each side of a multi-dimensional box). Since the spatial and temporal dimensions are orthogonal, an interval in the blue-magenta plane, , expands to:

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Where is the blue time distance and is the magenta squared time distance. But there is a problem with this formula. Time can’t be added to squared time anymore than a furlong can be added to an acre. A conversion to a common unit is needed. While a constant converting time into space isn’t possible, a conversion that is specific to a time coordinate can be found. The antiderivative of Eq. (4) describes a relation that allows us to convert our distance in time into a distance in space.

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Where is the constant of integration. Eq. (6) describes how time coordinates are converted into space coordinates at a given time, , in a reference frame.

An expression for as a function of is now needed to complete the metric using a common set of units (space).

A screenshot of a computer screen

Description automatically generated with low confidence

Figure The spacetime interval for the blue-magenta plane, , from point A to point B in quadratically expanding spacetime showing the line elements, and .

Figure 2 shows the relation between the components of the spacetime interval schematically. Using Eq. (2), the relation between and is:

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Where is the size of the spatial dimension at and is the size of the same spatial dimension at . From the schematic, we can see that the change in magenta space, , during this time interval is:

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|  |  | (7) |

The complete metric formula for a spacetime interval in the blue-magenta plane is:

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|  |  | (8) |

# Invariance

The metric formula of Eq. (8) can be analyzed to determine how distances change as the temporal coordinate axes rotate about the imaginary plane. The distance between two points is invariant if it doesn’t change as the temporal velocity changes.

Here we imagine yellow space made from the green-red plane as our (coordinate) reference frame. The magenta space in red-blue time is our moving (proper) frame. Geometrically, invariance can be expressed as:

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However, the observer is stationary, so the yellow space term drops out, leaving:

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If we assume that the end points are simultaneous such that and , and also assume that there is no change in the temporal velocity such that , then Eq. (7) reduces to:

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And knowing that , and extending our assumption about the simultaneity of the endpoints such that and , then an expression for can be found:

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|  |  | (9) |

Once is known, the angle, , as a function of spatial velocity can be calculated as:

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And from the angle between the time coordinates, , the temporal velocity as a function of spatial velocity is:

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If we further assume that the difference between and is so small as to be negligible, then the relation between coordinate time and proper time is approximated as:

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# Chromatic SpaceTime

Having established the invariance of distance (within the given assumptions), we now consider the permutations of space and time. One dimension of time is not interesting as there is no space. Two dimensions of time result in only one spatial dimension, so any discussion would be of limited value since we observe more than one dimension of space. If there were three dimensions of time, then we would observe three dimensions of space, one for each imaginary plane. This option has promise, so let us put a pin in it. If we had four dimensions of time, then we would observe six dimensions of space (again, one for each scalar product). But with six spatial dimensions, the intensity of light would fade much more quickly than it does with the observed inverse square law, so consideration of the properties of four (or more) dimensions of time is likely another academic exercise without more evidence of hidden dimensions.

Having considered the permutations, we will focus the discussion on a manifold with three imaginary dimensions of time and three real dimensions of space. We are going to label them as red time, , green time, and blue time, due to the way they combine to form distinct secondary dimensions. The product of green time and red time is yellow space, . The product of blue time and green time is cyan space, . The product of red time and blue time is magenta space, .

We can now describe the line elements for each of the three spacetime planes.

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These line elements combine to form the metric formula for six-dimensional chromatic spacetime:

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In the special case where all three temporal dimensions are aligned, forming an inertial reference frame, this formula can be simplified according to the conditions:

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In addition, the constants, and , can be combined into three-plane aggregates:

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With these assumptions and substitutions, the metric formula can be consolidated as:

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Where is the temporal coordinate of the observer, and is the temporal coordinate of the observed. This yields the metric tensor for a four-dimensional approximation of the six-dimensional manifold with a single evolution parameter.

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|  |  | (16) |
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# Luminous Distance

Because of the uniform mass of the progenitor star, type Ia Supernovae (SNe Ia) can be used to measure spacetime distances on cosmological scales. There are two observables from these stars that make this measurement possible: the change in photon’s wavelength, the redshift, tells us how space has expanded since the photon was emitted and the number of photons passing through a detector during a span of time from a known luminosity is a proxy for the spatial distance. We can use this information to test our hypothesis if we parameterize the metric tensor from Eq. (16) to use redshift. The relation between the redshift and scale factor is:

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|  |  | (17) |

Light follows a null geodesic in spacetime, so we set , and the and terms are also set to zero since the path of a photon is a line-of-sight measurement.

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This is our formula for comoving distance as a function of redshift. However, luminosity is a power measurement – a count of photons per area per time – so we must account for the fact that these photons are traveling faster now than when emitted and therefore the frequency of detection will be higher by a factor of .

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|  |  | (19) |

# Solutions to Einstein Field Equations