The Properties of a SIX-DIMENSIONAL, pseudo-Riemannian Manifold

# Abstract

Here we describe a six-dimensional, pseudo-Riemannian manifold with three imaginary dimensions of time and three real dimensions of space, we assign it some simple rules, and then describe the resulting properties. Any two temporal dimensions form an imaginary plane. Each spatial dimension is orthogonal to the imaginary plane and the extent of the spatial dimension is the dot product of the temporal coordinates. We then discuss the properties that this manifold possesses when projected onto the spatial dimensions and given a single evolution parameter. Here we demonstrate that the spatial projection of this manifold expands with time and even accelerates under its own power independent of any stress, energy, or momentum.

Some formal definitions are required for our discussion.

# Real and Imaginary

For the scope of this paper, real is that which can be measured directly: space, acceleration, and the square root of a positive area, for example, while imaginary is that which isn’t real: time, velocity, and the square root of a negative area for example.

# Time

Imagine a line. Give it a distinct origin. This is time.

# Squared Time

Time, by itself, is unremarkable, but if you have two dimensions of time in an imaginary plane, then some interesting properties emerge. A third dimension – the dimension of squared time – exists as the dot product of the temporal coordinates.

Because squared time is a secondary dimension derived from two primary dimensions, a chromatic index is used. For example, magenta squared time, , is the product of the red time coordinate, , and the blue time coordinate, . This can be expressed as:

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Where is the extent of the squared time dimension. Because the geometric definition of the dot product operator takes the norm of the vectors, we take some license here and employ the shorthand notation of instead to indicate that the basis vector has an imaginary component which cannot be ignored. This convention fixes the geometric definition so that it yields the same result as the algebraic definition when operating on two imaginary basis vectors.

From Eq. (1) we can see that the dimension of squared time expands quadratically as a function of time. That is, when both temporal coordinates advance at the same rate, all points will move away from each other with a constant acceleration. On this manifold, objects at rest accelerate.

Squared time has the additional property of being real, making it the only property of the manifold that can be measured directly.

# A Meter

Squared time and space are the same thing, but with different units. Since time is imaginary, a conversion to a real number is needed to make meaningful measurements.

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Where is the extent of the spatial dimension in some real unit that we’ll generically refer to as a meter, is the conversion factor between space and squared time.

A picture containing line chart

Description automatically generated

Figure 1 The relationship between time, and , and space, . The size of the spatial dimension is proportional to the dot product of the temporal coordinates. Space is always orthogonal to time.

# A Second

Hold a ruler in your hand. Space on this manifold can be measured with a ruler, however, being pseudo-Reimann, we must also measure the imaginary part. Now, imagine a ruler. Is it bigger or smaller than the one in your hand? Is it twice as big, half as big? Exactly how do we quantitatively compare something real with something imaginary?

Velocity relates space to time, so we could use a unit length and a given velocity to define a unit of time. But which velocity to use? All observers in all references frame must agree to use the same velocity when constructing the temporal coordinate axes or they will never agree upon the spacetime distance between two points and that would be a terrible choice for a coordinate system.

All points on the manifold have a well-defined tangent velocity: the partial derivative of the tangent space with respect to time. A unit of time, then, for the purpose of this paper, is the projection of an infinitesimal length, , through the tangent velocity, , and onto the dimension of time, . When two temporal axes are aligned, they form a reference frame and the time coordinates advance synchronously such that , and then Eq. (2) reduces to:

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From this, we can derive an expression for the tangent velocity:

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Where the constant acceleration of the manifold, is the constant of integration (that is, the tangent velocity at ), and is the tangential velocity at time, . From this relationship, we formalize our definition of a second:

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On the surface of this manifold meters are constant and seconds are variable and the length of a second decreases as a function of time. Note that this manifold doesn’t possess time symmetry.

# Tangent Space

At first blush, the fact that all points at a given time share the same tangent velocity would seem to make relative motion impossible. However, we can decompose the tangent vectors into smaller pieces to see if there’s some room for relativity. If space is described by:

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Then a pair of vectors can be constructed from the partial derivatives of our space with respect to the coordinate axes. We will use the red-yellow space as the reference frame (that is, a frame where ) and let define the coordinate time and let define the coordinate space.

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Where is the tangent vector in time (tangent velocity) and is the tangent vector in space. We can now see the outline of relativity in components that allow for any relative velocity, defined as , so long as the sum of the squares of the components is equal to the magnitude of the respective tangent vectors. Substituting Eq. (3) and into Eq. (5), we get:

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From this, the angle between the temporal axes can be expressed as a function of the relative velocity, , of the moving frame (magenta) to the reference frame (yellow)

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And from this we find the relationship between coordinate time, , and moving time, :

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The same method can be applied to the coordinate space. Substituting Eq. (3) and Eq. (7) into Eq. (6):

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From this relationship we can see that relative motion is possible so long as the sum of the squares of the component vectors adds up to the magnitude of the respective tangent vectors. That is, to balance the books, the faster a frame is moving, the slower and shorter it will appear when project onto a reference frame.

# Three-Dimensional Metric Formula

Distance can be calculated using the Pythagorean theorem.

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Where is the distance, is the metric tensor and is the difference between a set of coordinates in each dimension. As the spatial and temporal dimensions are orthogonal in this manifold, the interval, , simply expands to:

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Where is the spatial projection (equivalent) of the blue time length, , and is the magenta space length. Substituting Eq. (3), our definition of a second, we get the metric formula in terms of a measurable (albeit indirect) value, :

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However, there’s a problem with this formula. We have no way to directly measure . We cannot place a pin in the manifold at and use a ruler to measure how much space there is to because the point, , exists only in the past and rulers can only measure in the present. The schematic of Figure 2 illustrates that the only values that can be measured directly with a ruler (spatial coordinates) are and . We need to relate the coordinate distance, , to the line element, before this metric can be useful for measuring the real part of the manifold.

A screenshot of a computer screen

Description automatically generated with medium confidence

Figure The spacetime interval, , from to in quadratically expanding spacetime showing the line elements, and .

Using Eq. (2), the relation between and , as the manifold expands, is:

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Where is the spatial extent at and is the spatial extent at . Next, we will observe that the line element, , is the coordinate distance, , less one half of expansion (expressed as ):

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The metric formula for a spacetime distance using practical coordinates is then:

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# Six-Dimensional Metric Formula

We now consider the permutations of space and time. Two dimensions of time result in only one spatial dimension, so any discussion would be of limited value. Three dimensions of time result in three dimensions of space, one for each imaginary plane. This option agrees with the observed inverse-square law, so let us put a pin in it. If we had four dimensions of time, then we would observe six dimensions of space. We do not, so this option also shows little promise.

Having considered the permutations, we will focus the discussion on a manifold with three imaginary dimensions of time and three real dimensions of space. We are going to label them as red time, , green time, and blue time, due to the way they combine to form secondary dimensions. The product of green time and red time is yellow space, . The product of blue time and green time is cyan space, .

The additional line elements are:

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These line elements form the metric formula for six-dimensional spacetime:

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# Four-Dimensional Metric Formula

In the special case where all three temporal dimensions are aligned, forming an inertial reference frame in six dimensions, this formula can be simplified according to the conditions:

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In addition, the constants, and , can be combined into three-plane aggregates:

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With these assumptions and substitutions, the metric formula for a four-dimensional approximation of the six-dimensional manifold with a single evolution parameter, , is:

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Where is the spacetime distance between two events, and are the temporal coordinates of the two events, and , , and are the spatial distances between those two events.

# Solutions to Einstein Field Equations

# INitial Conditions

From this formula, we can see that there are three initial conditions which do not depend on the coordinates: the acceleration, , the initial velocity, , and the time (age) of the manifold at the time of observation, . It’s important to understand the scale of this manifold if we want to further explore the properties. The scale, however, depends heavily on these initial conditions.

If we chose a line-of-sign path along a null geodesic (that is, a path having and ) then the space distance between two events can be found by solving Eq. (16) for :

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This formula can be further simplified by encoding the time coordinates as:

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|  |  | (17) |

Using this formula and a selection of redshift values from Supernovae observations, we could extract a solution for our initial conditions using an optimization algorithm. We will imagine that we have performed this exercise and will use the results, with the assumption that they are very good guesses, as free variables to continue the discussion of the properties:

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Table 1 – The fiducial initial conditions.

# Curvature