The Properties of a Complex SIX-DIMENSIONAL Manifold

# Abstract

Here we describe a six-dimensional manifold with three imaginary dimensions of time and three real dimensions of space, we assign it some trivial rules, and then describe the resulting properties. The rules are thus: any two temporal dimensions form an imaginary plane. The spatial dimensions are orthogonal to the temporal ones and the extent of any spatial dimension is proportional to the time coordinates and the angle between the temporal dimensions. We then discuss the properties that this complex manifold possesses when projected onto the dimensions of space and given a single evolution parameter. We will demonstrate that the three-dimensional projection of this six-dimensional manifold expands spontaneously with time and even appears to accelerate under its own power.

Some formal definitions are required for our discussion.

# Time

Time is an imaginary dimension. Imagine a line, give it a basis vector, an origin, and assign it imaginary coordinates that are contravariant to the basis vector. This is time.

# Squared Time

Time, by itself, is unremarkable, but if you have two dimensions of time in an imaginary plane, then some interesting properties emerge. We can introduce a third dimension – the dimension of squared time – that is the scalar product of two imaginary coordinates. The size of this secondary dimension is:

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Where is magenta space in units of squared time. Because the spatial dimension is secondary to the primary temporal dimensions, a chromatic index is used instead of a numeric one. For example, magenta is the product of red and blue. , is the coordinate in red time, , is the coordinate in blue time, and is the angle between the red and blue coordinate axes in the imaginary plane. As a convention, red time will be the coordinate axis of the observer, and blue time will be the coordinate axis (proper time) of the observed.

The magenta squared time axis is always orthogonal to the blue time axis. The observer has no need to locate themselves in space, only observed object need space. However, a fiducial time axis can be parallel to (aligned with) the observer’s time axis. This alignment defines an inertial reference frame and gives the observer a benchmark axis with which to compare all other orientations. The orientation of the temporal axes, , is equal to the temporal velocity, . For an inertial reference system, is zero and the temporal velocity is one.

From Eq. (1) we see that the size of the spatial dimension of the observed object not only depends on the time coordinates, but on the temporal velocity as well. For example, the magenta squared time coordinate axis will contract as the temporal velocity between red time and blue time decreases and will vanish entirely when the coordinate axes are orthogonal (e.g., ).

Squared time has the additional property of being real. Because of this quality, the squared time projection of a complex manifold is a real submanifold and we can explore how this distinct terrain changes as the imaginary coordinates evolve.

# Space

Squared time and space are the same dimension, but with different units. One cannot measure an imaginary unit like time with a real measuring stick. If we want to discuss the properties of this manifold in real numbers, then a conversion is needed.

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Where is the extent of the magenta spatial dimension in some practical unit (e.g., ), is the conversion factor between units of space and squared time (e.g., ).

# Inertial Velocity and Acceleration

In the special case of an inertial reference frame, the temporal axes are aligned and can be combined into single evolution parameter, . Under these conditions, time advances at the same rate and the trigonometric term of Eq. (2) is unity (, ) and we can write:

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This tells us how space changes with squared time an inertial reference frame in an empty manifold:

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Where is the acceleration of the expansion of space and is the tangent plane to the manifold at time . is an important feature of this manifold. Eq. (4) tells us that constant acceleration is manifested at every point in space independent of the coordinates. The first and most important property of this manifold is that all objects in an inertial frame will spontaneously drift away from each other at a constant rate of as the time coordinate advances. That is, objects at rest accelerate.

# Distance

The distance between any two sets of coordinates is invariant. It is a geometrical thing that is independent of any set of coordinates and any motion. The distance can be calculated using the Pythagorean theorem.

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Where is the distance, is the metric tensor and is the difference between a set of coordinates in each dimension. Since the spatial and temporal dimensions are orthogonal, an interval in the blue-magenta plane expands to:

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Where is the blue time distance and is the magenta space distance. We can use Eq. (4) integrated over time to find a relation that allows us to convert to a common unit:

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Where is the tangent plane of the manifold at time, , and is the constant of integration. Eq. (6) describes how time coordinates are converted into space coordinates at a given time. Note that while time and velocity are imaginary, space and acceleration are real.

Now we need an expression for as a function of .

A screenshot of a computer screen

Description automatically generated with low confidence

Figure The spacetime interval for the blue-magenta plane, , from point A to point B in quadratically expanding spacetime showing the line elements, and .

Figure 1 shows the relationship between the components of the spacetime interval schematically. The relation between and is:

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Where is the size of the spatial dimension at and is the size of the same dimension at . From the schematic, we can see that the change in magenta space, , during this time interval is:

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The complete metric formula for a spacetime interval in the blue-magenta plane is:

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# Chromatic SpaceTime

Now let us consider the permutations of space and time. One dimension of time is not interesting as there is no space. Two dimensions of time result in only one spatial dimension, so any discussion would be of limited value since we observe more than one dimension of space. If there were three dimensions of time, then we would observe three dimensions of space, one for each imaginary plane. This option has promise, so let us put a pin in it. If we had four dimensions of time, then we would observe six dimensions of space (again, one for each scalar product). But with six spatial dimensions, the intensity of light would fade much more quickly than it does with the observed inverse square law, so consideration of the properties of four (or more) dimensions of time is likely another academic exercise without more evidence of hidden dimensions.

Having considered the permutations, we will focus the discussion on a manifold with three imaginary dimensions of time and three real dimensions of space. We are going to label them as red time, , green time, and blue time, due to the way they combine to form distinct secondary dimensions. The product of green time and red time is yellow space, . The product of blue time and green time is cyan space, . The product of red time and blue time is magenta space, .

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We can now describe the line elements for each of the three spacetime planes.

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These line elements combine to form the metric formula for six-dimensional chromatic spacetime:

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In the special case where all three temporal dimensions are aligned, forming an inertial reference frame, this formula can be simplified according to the conditions:

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In addition, the constants, and , can be combined into three-plane aggregates:

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With these assumptions and substitutions, the metric formula can be consolidated as:

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Where is the temporal coordinate of the observer, and is the shared coordinate of the temporal dimensions of the observed. This yields the metric tensor for a four-dimensional approximation of six-dimensional spacetime with a single evolution parameter.

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|  |  | (14) |
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# Invariance

# Einstein Field Equations

# Luminous Distance

Because of the uniform mass of the progenitor star, type Ia Supernovae (SNe Ia) can be used to measure spacetime distances on cosmological scales. There are two observables from these stars that make this measurement possible: the change in photon’s wavelength, the redshift, tells us how space has expanded since the photon was emitted and the number of photons passing through a detector during a span of time from a known luminosity is a proxy for the spatial distance. We can use this information to test our hypothesis if we parameterize the metric tensor from Eq. (14) to use redshift. The relation between the redshift and scale factor is:

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Light follows a null geodesic in spacetime, so we set , and the and terms are also set to zero since the path of a photon is a line-of-sight measurement.

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This is our formula for comoving distance as a function of redshift. However, luminosity is a power measurement – a count of photons per area per time – so we must account for the fact that these photons are traveling faster now than when emitted and therefore the frequency of detection will be lower by a factor of .

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