The Properties of a SIX-DIMENSIONAL, pseudo-Riemannian Manifold

# Abstract

Here we describe a six-dimensional, pseudo-Riemannian manifold with three imaginary dimensions of time and three real dimensions of space, we assign it some simple rules, and then describe the resulting properties. Any two temporal dimensions form an imaginary plane. The spatial dimension is orthogonal to the imaginary plane and the extent of the spatial dimension is proportional to the time coordinates and the angle between the temporal dimensions – the temporal velocity – in the imaginary plane. We then discuss the properties that this manifold possesses when projected onto the spatial dimensions and given a single evolution parameter. Here we demonstrate that the spatial projection of this manifold expands spontaneously with time and even appears to accelerate under its own power independent of any stress, energy, or momentum.

Some formal definitions are required for our discussion.

# Real and Imaginary

For the scope of this paper, real that which can be measured directly: space and acceleration for example, while imaginary is that which can be imagined, but cannot be measured directly with any instrument: time, velocity, and the square root of a negative number for example.

# Time

Imagine a line. Give it a distinct origin. This is time.

# Squared Time

Time, by itself, is unremarkable, but if you have two dimensions of time in an imaginary plane, then some interesting properties emerge. A third dimension – the dimension of squared time – exists as the product of two imaginary dimensions. The relationship between these dimensions is:

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Where is the extent of the squared time dimension. Because squared time is a secondary dimension derived from two primary dimensions, a chromatic index is used. For example, magenta squared time is the product of red time and blue time. , is the coordinate in red time, , is the coordinate in blue time, and is the temporal velocity. The temporal velocity is also the angle between the temporal axes, , and will be used interchangeably.

From Eq. (1) we can see that the dimension of squared time expands quadratically as a function of time. When both temporal coordinates advance at the same rate, all points will move away from each other with a constant acceleration. Objects at rest in this manifold will accelerate.

Squared time has the additional property of being real.

# A Meter

Squared time and space are the same dimension, but with different units (basis vectors). Since time is imaginary, a conversion to a real number is needed to make meaningful measurements.

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Where is the extent of the spatial dimension in some real unit that we’ll generically refer to as a meter, is the conversion factor between units of space and units of squared time.

A picture containing line chart

Description automatically generated

Figure 1 A schematic of the relationship between time, and , and space, . The size of the spatial dimension is proportional to the temporal coordinates and velocity (). Space is always orthogonal to time.

# A Second

Hold a ruler in your hand. Space can be measured with a ruler in this pseudo-Reimann manifold. Being pseudo-Reimann, we must also measure the imaginary part. Now, imagine a ruler. Is it bigger or smaller than the one in your hand? Is it twice as big, half as big? Exactly how do we quantitatively compare something real with something imaginary?

Velocity relates space to time, so we could use a unit length and a given velocity to define a unit of time. But which velocity to use? All observers in all references frame must agree to use the same velocity when constructing the temporal coordinate axes or they will be unable to agree upon the size of any physical object.

On a manifold of constant acceleration, all points share the same tangent space. A unit of time, then, for the purpose of this paper, is the projection of a unit length, , through the tangent velocity, , and onto the dimension of time, . In the case where two temporal axes are aligned (at rest) such that the temporal velocity, is unity, and the time coordinates advance synchronously such that , Eq. (2) reduces to:

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From this, we can derive an expression for the tangent velocity (that is, the partial derivative of the tangent space with respect to time):

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Where the constant acceleration of the manifold, is the constant of integration, and is the tangential velocity (space per time). From this relationship, we formalize our definition of a second:

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|  |  | (3) |

Note that on the surface of this manifold, using this coordinate system, that time accelerates. Meters are constant, seconds are variable. A second when the manifold is old is going to be longer than a second when the manifold is new.

# Tangent Space

All points on the surface of the manifold, at a given time, share the same tangent velocity. At first blush, this would seem to make relative motion impossible. However, we can decompose the tangent space into smaller pieces to see if there’s some room for relative movement. If the tangent space is defined as:

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|  |  | (4) |

Then a plane can be constructed from the partial derivatives along the coordinate axes of the observer. We will use the red-yellow plane as the reference plane of the observer, that is, the coordinate axes and blue-magenta for the tangent plane.

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|  |  | (4) |
|  |  | (5) |

Where is the time-like tangent vector, and is the space-like tangent vector. We can now see the outline of relative motion in that these components are not fixed but allow for any value so long as the sum of the vector components is equal to the norm of the respective tangent vectors. Substituting Eq. (3) and , we get:

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From this, we can extract a formula for the relative velocity of the observer to the tangent plane:

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And from this we find the relation between coordinate time and tangent time:

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# The Metric

Distance can be calculated using the Pythagorean theorem.

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Where is the distance, is the metric tensor and is the difference between a set of coordinates in each dimension. As the spatial and temporal dimensions are orthogonal, the interval, , expands to:

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Where is the blue time length and is the magenta space length. These values are not easily obtained from observation, so substitutions to observable coordinates are needed. The tangent velocity describes the relation between and , so now we need an expression for as a function of the change in spatial coordinates, and .

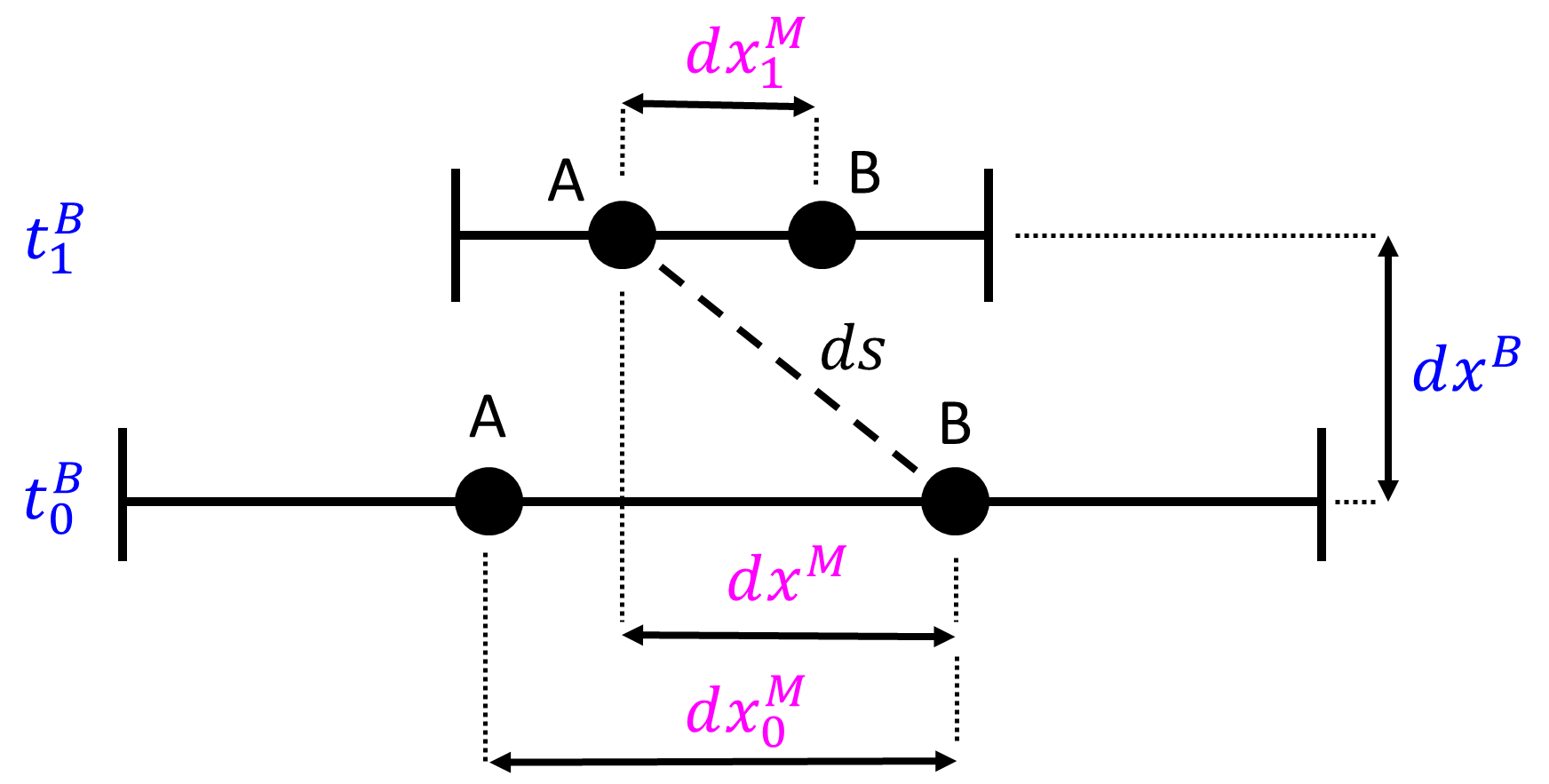


Figure The spacetime interval, , from point A to point B in quadratically expanding spacetime showing the line elements, and .

Figure 2 shows the relation between the components of the spacetime interval schematically. Using Eq. (2), the relation between and as space expands is:

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Where is the size of the spatial dimension at and is the size of the same spatial dimension at . From the schematic, we can see that the change in magenta space, , during the time interval, to , is:

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|  |  | (7) |

The complete metric formula for a spacetime interval is then:

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|  |  | (8) |

# Six-Dimensional SpaceTime

We now consider the permutations of space and time. Two dimensions of time result in only one spatial dimension, so any discussion would be of limited value since we observe more than one dimension of space. Three dimensions of time result in three dimensions of space, one for each imaginary plane. This option agrees with the observed inverse-square law, so let us put a pin in it. If we had four dimensions of time, then we would observe six dimensions of space. We don’t observe six spatial dimensions, so this option shows little promise.

Having considered the permutations, we will focus the discussion on a manifold with three imaginary dimensions of time and three real dimensions of space. We are going to label them as red time, , green time, and blue time, due to the way they combine to form secondary dimensions. The product of green time and red time is yellow space, . The product of blue time and green time is cyan space, . The product of red time and blue time is magenta space, .

The line elements for each of the three spacetime planes are:

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These line elements combine to form the metric formula for six-dimensional spacetime:

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# Four-Dimensional Spacetime

In the special case where all three temporal dimensions are aligned, forming an inertial reference frame in six dimensions, this formula can be simplified according to the conditions:

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In addition, the constants, and , can be combined into three-plane aggregates:

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With these assumptions and substitutions, the metric formula reduces to:

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Where is the temporal coordinate of the observer, and is the temporal coordinate of the observed. This yields the metric tensor for a four-dimensional approximation of the six-dimensional manifold with a single evolution parameter.

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|  |  | (14) |
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# Age ANd Acceleration

# Solutions to Einstein Field Equations

# Curvature