The Properties of a SIX-DIMENSIONAL, pseudo-Riemannian Manifold

# Abstract

Here we describe a six-dimensional, pseudo-Riemannian manifold with three imaginary dimensions of time and three real dimensions of space, we assign it some simple rules, and then describe the resulting properties. Any two temporal dimensions form an imaginary plane. Each spatial dimension is orthogonal to the imaginary plane and the extent of the spatial dimension is the dot product of the temporal coordinates. We then discuss the properties that this manifold possesses when projected onto the spatial dimensions and given a single evolution parameter. Here we demonstrate that the spatial projection of this manifold expands with time and even accelerates under its own power independent of any stress, energy, or momentum contained therein.

Some formal definitions are required for our discussion.

# Real and Imaginary

For the scope of this paper, real is that which can be measured directly: space, acceleration, and the square root of a positive area, for example, while imaginary is that which isn’t real and, thus, can only be inferred: time, velocity, and the square root of a negative area, for example.

# Time

Imagine a line. Give it a distinct origin. This is time.

# Squared Time

Time, by itself, is unremarkable, but if you have two dimensions of time in an imaginary plane, then some interesting properties emerge. A third dimension – the dimension of squared time – exists as the dot product of the temporal coordinates.

Because squared time is a secondary dimension derived from two primary dimensions, a chromatic index is used. For example, magenta squared time, , is the product of the red time coordinate, , and the blue time coordinate, . This can be expressed as:

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Where is the extent of the squared time dimension and is the angle between the temporal axes, and is also the temporal velocity, . Because the geometric definition of the dot product operator takes the norm of the vectors, we take some license here and employ the shorthand notation of instead to indicate that the basis vector has an imaginary component which cannot be ignored by the norm operator. This convention fixes the geometric definition so that it yields the same result as the algebraic definition when operating on two imaginary basis vectors.

From Eq. (1) we can see that the dimension of squared time expands quadratically as a function of time. That is, when both temporal coordinates advance at the same rate, all points will move away from each other with a constant acceleration. On this manifold, objects at rest accelerate.

Squared time has the additional property of being real, making it the only dimension that can be measured directly.

# A Meter

Squared time and space are the same thing, but with different units. Since time is imaginary, a conversion to a real number is needed to make meaningful measurements.

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Where is the extent of the spatial dimension in some real unit that we’ll generically refer to as a meter, is the conversion factor between space and squared time. Space, then, is a negative area and time is the square root of that negative area.

A picture containing line chart

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Figure 1 The relationship between red time, , blue time, , and magenta space, . The size of the spatial dimension is proportional to the dot product of the temporal coordinates. Space is always orthogonal to time.

# A Second

Hold a ruler in your hand. Space on this manifold can be measured with a ruler, however, being pseudo-Reimann, we must also measure the imaginary part. Now, imagine a ruler. Is it bigger or smaller than the one in your hand? Is it twice as big, half as big? Exactly how do we objectively compare something real with something imaginary?

Velocity relates space to time, so we could use a unit length and a given velocity to define a unit of time. But which velocity to use? All observers in all references frame must agree to use the same velocity when constructing the temporal coordinate axes or they will never agree upon the spacetime distance between two points.

All points on the manifold have a well-defined tangent velocity: the partial derivative of the tangent space with respect to time. A second, then, for the purpose of this paper, is the projection of an infinitesimal length, , through the tangent velocity, , and onto the dimension of time, .

When two temporal axes are aligned, they form a reference frame, and the time coordinates advance synchronously such that . Eq. (2) then reduces to:

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|  |  | (3) |

From this, an observer in the reference frame can derive an expression for the tangent velocity on the surfaced of this manifold which will be the same for every frame at a given time:

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Where is the constant acceleration of the manifold, is the constant of integration (that is, the tangent velocity at ), and is the tangential velocity. From this relationship, we formalize our definition of a second:

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|  |  | (6) |

Where a second, , is the unit of time (basis vector) of our inferred temporal axis. With this choice of a coordinate system, meters are constant, seconds are variable and the length of a second decreases as a function of time. Note that this manifold doesn’t possess time symmetry.

# Tangent Space

At first blush, the fact that all points at a given time share the same tangent velocity would seem to make relative motion impossible. However, we can decompose the tangent vectors into smaller pieces to see if there’s some room for relativity. If the space of a moving object is described by:

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Where is the equivalent of in units of meters, then a pair of vectors can be constructed from the partial derivatives of the tangent space with respect to the coordinate axes. We will use the red-yellow plane as the reference frame (that is, a frame where ) and let define the coordinate time and let define the coordinate space.

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Where is the tangent vector along the temporal coordinate axis (tangent velocity) and is the tangent vector along the spatial coordinate axis. We can now see the outline of relativity in components that allow for any relative velocity, defined as , so long as the sum of the squares of the sides is equal to the square of the hypotenuse, so to speak. Substituting Eq. (6) and into Eq. (8), we get:

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From this, the angle between the temporal axes can be expressed as a function of the relative velocity,

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And from this we find the relationship between coordinate time, , and moving time, :

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The same method can be applied to the tangent vector along the spatial coordinate axis. Substituting Eq. (6) and Eq. (10) into Eq. (9):

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From this relationship we can see that relative motion is possible so long as the sum of the squares of the component vectors adds up to the squares of the respective tangent vectors. Stated more plainly: the faster a frame is moving, the slower and shorter it will appear when project onto a reference frame.

# Three-Dimensional Metric Formula

Distance can be calculated using the Pythagorean theorem.

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Where is the distance, is the metric tensor and is the difference between a set of coordinates in each dimension. As the spatial and temporal dimensions are orthogonal in this manifold, the interval, , simply expands to:

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Where is the spatial projection (equivalent) of the blue time length, , and is the magenta space length. Substituting Eq. (6), we get the metric formula in terms of a measurable (albeit indirect) value, :

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Where is the constant time of the observation, and is the arbitrary time of some event. However, there is a problem with this formula. We have no way to directly measure . We cannot place a pin in the manifold at and use a ruler to measure how much space there is to because the point, , exists only in the past and rulers can only measure in the present. The schematic of Figure 2 illustrates that the only values that can be measured directly with a ruler (spatial coordinates) are and . We need to relate the coordinate distance, , to the line element, before this metric can be useful for measuring the real part of the manifold.

A screenshot of a computer screen

Description automatically generated with medium confidence

Figure The spacetime interval, , from to in quadratically expanding spacetime showing the line elements, and .

Using Eq. (2), the relation between and , as the manifold expands, is:

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Where is the spatial extent at and is the spatial extent at . Next, we will observe that the line element, , is the coordinate distance, , less one half of expansion (expressed as ):

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The metric formula for a spacetime distance using practical coordinates is then:

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# Six-Dimensional Metric Formula

We now consider the permutations of space and time. Two dimensions of time result in only one spatial dimension, so any discussion would be of limited value. Three dimensions of time result in three dimensions of space, one for each imaginary plane. This option agrees with the observed inverse-square law, so let us put a pin in it. If we had four dimensions of time, then we would observe six dimensions of space. We do not, so this option also shows little promise.

Having considered the permutations, we will focus the discussion on a manifold with three imaginary dimensions of time and three real dimensions of space. We are going to label them as red time, , green time, and blue time, due to the way they combine to form secondary dimensions. The product of green time and red time is yellow space, . The product of blue time and green time is cyan space, .

The additional line elements are:

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These line elements form the metric formula for six-dimensional spacetime:

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# Four-Dimensional Metric Formula

In the special case where all three temporal dimensions are aligned, forming an inertial reference frame in six dimensions, this formula can be simplified according to the conditions:

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In addition, the constants, and , can be combined into three-plane aggregates:

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With these assumptions and substitutions, the metric formula for a four-dimensional approximation of the six-dimensional manifold with a single evolution parameter, , is:

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Where is the spacetime distance between two events, and are the temporal coordinates of the two events, and , , and are the spatial distances between those two events.

# INitial Conditions

From Eq. (19) we can see that there are three initial conditions for quadratically expanding space (QES) which do not depend on the coordinates: the acceleration, , the initial tangent velocity, , and the age of the manifold at the time of observation, .

If we chose a line-of-sign path along a null geodesic (that is, a path having and ) then the spatial distance between two events can be found by solving Eq. (19) for :

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This formula can be further simplified by encoding the time coordinates as:

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|  |  | (20) |

Where , the redshift, is the change in a unit length (wavelength) from to . This redshift value is encoded in photons, which travel along the null geodesic, making it possible to use the photons from a known source (of luminosity) as proxies for distance markers.

Chart

Description automatically generated

Figure 3 - Top: The distances to a selection of 482 SNe Ia supernovae (green) and the distance predicted by the quadratically expandig space (QES) metric formula (red) and, for comparison, the distance predicted by the FLRW metric formula (blue). Bottom: Errors for each of the data points from the predicted values in Gpc.

Using the combined data from (Conley et al. 2011), (Rodney et al. 2012), (Jones et al. 2013), (Rodney et al. 2015) and the parameters from (Rodney, et al. 2015) to normalize the sets, we can extract the initial conditions using a minimization algorithm on the aggregate of the error values (chi-square minimization). A of tells us that the SNe Ia provide, not only an excellent match to the model, but also a reasonable set of initial conditions with which we can continue our discussion.

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Table 1 – The initial conditions.

# Solutions to Einstein Field Equations

The Einstein Field Equations describes additional properties of this manifold. Before we can use them, we must solve them. The first step is to derive the metric tensor for Eq. (19). For this tensor, we assume that the terminus of the geodesic, , is constant and is arbitrary and allowed to vary as time, .

Where is a function of the geometry. For geometries with elliptical curvature, . For flat geometries, . For hyperbolic geometries, .The

# Curvature